AMERICAN DOCTORIAL DISSERTATIONS.

F. R. MORRIS, "Classification of involutory cubic space transformations," *University of California Publications in Mathematics*, vol. 1, no. 11, February, 1920, pp. 223–240. (University of California, 1918).

A. R. WILLIAMS, "On a birational transformation connected with a pencil of cubics," *University of California Publications in Mathematics*, vol. 1, no. 10, February, 1920, pp. 211–222. (University of California, 1916).

PROBLEMS AND SOLUTIONS.

EDITED BY B. F. FINKEL AND OTTO DUNKEL.

[Send all communications about problems and solutions to B. F. Finkel, Springfield, Mo.]

PROBLEMS FOR SOLUTION.

[N.B. The editorial work of this department would be greatly facilitated if, on sending in problems, the proposers would also enclose their solutions—*when they have them*. If a problem proposed is not original the proposer is requested *invariably* to state the fact and to give an exact reference to the source.]

2834. Proposed by OTTO DUNKEL, Washington University.

In any triangle ABC let M and N be, respectively, the points in which the median and the bisector of the angle at A meet the side BC, Q and P the points in which the perpendicular at N to NA meets MA and BA, respectively, and O the point in which the perpendicular at P to BA meets AN produced. Prove that the straight line QO is perpendicular to BC and the similar theorem for the external bisector of the angle at A.

This proposition shows the relation between two constructions for the center of curvature O of a conic for which B and C are foci and A is a point of the conic. (The figure also gives an easy proof of the Law of Tangents for triangles compare *1920*, 53–54. See also *1920*, 226.)

2835. Proposed by J. L. RILEY, Stephenville, Texas.

If x, y, z, u are finite, and not all zero, and satisfy the equations

$$x = by + cz + du, \qquad y = ax + cz + du, \qquad z = ax + by + du, \qquad u = ax + by + cz,$$

and if none of the quantities a, b, c, d, have the value -1, then will

$$\frac{a}{1+a} + \frac{b}{1+b} + \frac{c}{1+c} + \frac{d}{1+d} = 1.$$

2836. Proposed by W. V. N. GARRETSON, Rutgers College.

A ladder 40 feet long rests with one end on the ground against the foot of a building and the other end against the side of a second building directly across the street from the first. A second ladder 25 feet long inclines in a similar manner from the foot of the second building against the side of the first building, the two ladders crossing at a point 15 feet above the ground. How wide is the street?

2837. Proposed by B. F. FINKEL, Drury College.

Assuming v to be the velocity of sound; $\pm u_s$ the velocity of S, the source of sound, and n its frequency; $\pm u_r$ the velocity of R, the receiver; and $\pm w$ the velocity of M, the medium, discuss fully Döppler's Principle for the apparent frequency n'. The double signs are used to indicate that the discussion is to include the cases when source and receiver are approaching and when separating and the same consideration with reference to the medium. Limiting cases are especially desirable; e.g., when $w = 0$, $u_s = 0$, and $-u_r = 2v$; $w = 0$, $+u_s = v$, and $u_r = 0$; etc.

2838. "A rope is supposed to be hung over a wheel fixed to the roof of a building; at one end of the rope a weight is fixed, which exactly counterbalances a monkey which is hanging on to the other end. Suppose that the monkey begins to climb the rope, what will be the result?"

This problem was invented by Lewis Carroll in December, 1893 (S. D. Collingwood, *The Life and Letters of Lewis Carroll* (*Rev. C. L. Dodgson*), New York, 1899, pp. 317–318) and in his diary he remarked: "Got Professor Clifton's answer [R. B. Clifton, professor of physics at Oxford] to the 'Monkey and Weight Problem.' It is very curious, the different views taken by good mathematicians. Price [Bartholomew Price, professor of physics at Oxford] says that the weight goes *up* with increasing velocity; Clifton (and Harcourt [A. G. Vernon-Harcourt, professor of chemistry at Oxford]) that it goes *up*, at the same rate as the monkey; while Sampson [probably E. F. Sampson, lecturer, tutor and censor of Christ Church, Oxford] says that it goes *down*." Yet another solution by Rev. A. Brook is given on page 268 of *The Lewis Carroll Picture Book* . . . edited by S. D. Collingwood (London, 1899), namely that "the weight remains stationary."

The problem has been recently discussed in *School Science and Mathematics*, volume 17, December, 1917, p. 821; volume 19, December, 1919, p. 815; and volume 20, February, 1920, pp. 172–173. The editors of the MONTHLY invite mathematical solutions of the problem.

2839. By translating the steps of the construction of a regular pentagon from plane geometry into algebra show that one of the fifth roots of unity is equal to

$$\tfrac{1}{4}(\sqrt{5}-1)+\frac{i}{4}\sqrt{10+2\sqrt{5}}.$$

(This problem is proposed for solution in Wilczynski and Slaught, *College Algebra with Applications*, Boston, 1916, p. 193.)

2840. Proposed by NORMAN ANNING, University of Maine.

It is observed in a table of values of

$$\log_{10}(\text{colog}_{10}\, x)$$

that second differences are zero for values of x in the neighborhood of 0.37. Prove that this must be the case. (Cf. Chappell's *Five-Figure Mathematical Tables*, Edinburgh, 1915, p. 180.)

2841. Proposed by WILLIAM HOOVER, Columbus, Ohio.

The mixed number,

$$9\frac{49}{64}=9+\frac{49}{64}=3^2+\frac{7^2}{8^2},$$

is of the type form

$$k^2+\frac{(2k+1)^2}{(2k+2)^2};$$

how may the *forms* of the terms of the fractional part be determined *deductively?*

Generally required that

$$k^2+\frac{\{\varphi_1(k)\}^2}{\{\varphi_2(k)\}^2}$$

be a perfect square; show how $\varphi_1(k)$ and $\varphi_2(k)$ may be found.

2842. Proposed by H. S. UHLER, Yale University.

Express explicitly the following sextic in x as the product of a quadratic and a biquadratic:

$$3x^6-6k_1x^5+(7k_1^2-9k_2^2)x^4-2(2k_1^3-4k_1k_2^2-3k_3^3)x^3+[(k_1^2-k_2^2)^2-9k_1k_3^3]x^2$$
$$-(k_1^2-2k_2^2)(k_1k_2^2-9k_3^3)x+(k_1^2-3k_2^2)(k_2^4-3k_1k_3^3).$$

SOLUTIONS OF PROBLEMS.

2746 [1919, 72]. Proposed by S. A. COREY, Des Moines, Iowa.

Establish the following algebraic identity without actually performing the indicated operations:

$$2(t_1t_2+c_1t_3t_4+c_2t_5t_6+c_1c_2t_7t_8)(r_1r_2+c_1r_3r_4+c_2r_5r_6+c_1c_2r_7r_8)$$
$$=(r_1t_1-c_1r_3t_3-c_2r_5t_5+c_1c_2r_7t_7)(r_2t_2-c_1r_4t_4-c_2r_6t_6+c_1c_2r_8t_8)$$
$$+(r_1t_2-c_1r_3t_4-c_2r_5t_6+c_1c_2r_7t_8)(r_2t_1-c_1r_4t_3-c_2r_6t_5+c_1c_2r_8t_7)$$
$$+c_1(r_1t_3+r_3t_1-c_2r_5t_7-c_2r_7t_5)(r_2t_4+r_4t_2-c_2r_6t_8-c_2r_8t_6)$$